

# Social Mobility and Redistributive Politics

Piketty (QJE, 1995)

summary by N. Antić

- Most people believe that unequal opportunities which are beyond individual control are a bad thing
- Thus, they believe that government should intervene to provide ex-ante equal opportunity for everyone
  - In the redistribution setting, all voters agree that redistribution is good as long as it does not affect incentives to work
- If voters agree on the objective function, how can voting behaviour be so different
- Stylized Fact: Table summarizes voting behaviour (percentage who vote for left-wing parties) in six countries as a function of mobility experience

	Low Income Parents	High Income Parents
Low Income Individual	72%	49%
High Income Individual	38%	24%

- Piketty (1995) reconciles these observations in a model of dynastic learning
  - The mobility experience of individuals will impact on their belief about the cost of redistributive taxes for society

## Model and Notation

- Discrete time,  $t = 1, 2, \dots$  and a continuum of infinitely-lived dynasties  $i \in I \stackrel{\text{def}}{=} [0, 1]$
- Pretax income of generation  $t$  of dynasty  $i$  is  $y_t^i = y_x$  with  $x \in \{0, 1\}$  and  $y_1 > y_0 > 0$
- Agent  $i_t$  chooses effort level  $e_t^i$  in period  $t$  and gets payoff:

$$U_t^i(y_t^i, e_t^i) = (1 - \tau_t) y_{it} - \frac{(e_t^i)^2}{2a} + \tau_t Y_t,$$

where  $\tau_t$  and  $Y_t$  are the tax rate and aggregate income in period  $t$ , and  $a > 0$  is a parameter

- Income of  $i_t$  depends on parents' income and effort
  - $\Pr(y_t^i = y_1 | e_t^i, y_{t-1}^i = y_x) = \pi_x + \theta e_t^i$
- Timing of actions during a period:
  1. Given tax level  $\tau_t$  agent  $i_t$  chooses effort level  $e_t^i$
  2. Income shock is realized
  3. Agents vote on  $\tau_{t+1}$
- An agent facing taxes  $\tau_t$  and parameters  $(\theta, \pi_0, \pi_1)$  would choose effort:

$$\begin{aligned} e &= \arg \max_{e \geq 0} (\pi_x + \theta e) [(1 - \tau_t) y_1 + \tau_t Y_t] \\ &\quad + (1 - \pi_x - \theta e) [(1 - \tau_t) y_1 + \tau_t Y_t] - \frac{e^2}{2a} \\ &= \arg \max_{e \geq 0} (\pi_x + \theta e) (1 - \tau_t) y_1 - (\pi_x + \theta e) (1 - \tau_t) y_0 \\ &\quad - \frac{e^2}{2a} + \tau_t Y_t + (1 - \tau_t) y_0 \\ &= \arg \max_{e \geq 0} (\pi_x + \theta e) (1 - \tau_t) (y_1 - y_0) - \frac{e^2}{2a} \end{aligned}$$

The FOC is:

$$\theta (1 - \tau_t) (y_1 - y_0) - \frac{e}{a} = 0,$$

so that the optimal effort is a function of  $\tau_t$  and  $\theta$ :

$$e(\tau_t, \theta) = a\theta (1 - \tau_t) (y_1 - y_0).$$

Note that the SOC is satisfied.

- Partition  $[0, 1]$  into various types of voters:

	$y_{t-1}^i = y_0$	$y_{t-1}^i = y_1$
$y_t^i = y_0$	$SL_t$	$DM_t$
$y_t^i = y_1$	$UM_t$	$SH_t$

- $SL_t$  is the set of "stable low-income" agents at time  $t$
- $UM_t$  is the set of "upwardly mobile" agents
- $DM_t$  is the set of "downwardly mobile" agents
- $SH_t$  is the set of "stable high-income" agents

- Let  $L_t = SL_t \cup UM_t$  and  $H_t = DM_t \cup SH_t = [0, 1] \setminus L_t$

- Voting preferences are identical, all agents maximize

$$V_{t+1} = \int_{i \in L_{t+1}} U_i^{t+1} di$$

- Given the above optimal effort function and parameters  $(\theta, \pi_0, \pi_1)$ , the most preferred tax rate that player  $i_t$  would be found by solving  $\tau_{t+1} = \arg \max_{\tau \geq 0} \int_{L_{t+1}} U_{t+1}^i di$ , where:

$$\begin{aligned} \int_{L_{t+1}} U_{t+1}^i &= (\pi_0 + \theta e(\tau, \theta)) (1 - \tau) y_1 + \tau y_0 \\ &\quad + (1 - \pi_0 - \theta e(\tau, \theta)) (1 - \tau) y_0 - \frac{e(\tau, \theta)^2}{2a} \\ &\quad + \tau \left( \begin{array}{c} \pi_0 \lambda(L_{t+1}) \\ + \pi_1 \lambda(H_{t+1}) + \theta e(\tau, \theta) \end{array} \right) (y_1 - y_0) \\ &= (1 - \tau) y_0 - \frac{a\theta^2 (1 - \tau)^2 (y_1 - y_0)^2}{2} \\ &\quad + (\pi_0 + a\theta^2 (1 - \tau) (y_1 - y_0)) (1 - \tau) (y_1 - y_0) \\ &\quad + \tau (\pi_0 \lambda(L_{t+1}) + \pi_1 \lambda(H_{t+1})) (y_1 - y_0) \\ &\quad + a\theta^2 \tau (1 - \tau) (y_1 - y_0)^2 + \tau y_0 \end{aligned}$$

- The FOC for this problem is:

$$\begin{aligned} 0 &= -y_0 + y_0 - \pi_0 (y_1 - y_0) - 2a\theta^2 (y_1 - y_0)^2 (1 - \tau) \\ &\quad + a\theta^2 (y_1 - y_0)^2 (1 - \tau) + \pi_0 \lambda(L_{t+1}) (y_1 - y_0) \\ &\quad + \pi_1 \lambda(H_{t+1}) (y_1 - y_0) + a\theta^2 (y_1 - y_0)^2 (1 - 2\tau) \\ 0 &= (\pi_1 - \pi_0) \lambda(H_{t+1}) + a\theta^2 (y_1 - y_0) \tau - a\theta^2 2\tau (y_1 - y_0), \\ &\Rightarrow \tau = \frac{(\pi_1 - \pi_0) \lambda(H_{t+1})}{a\theta^2 (y_1 - y_0)}. \end{aligned}$$

- Agent with beliefs  $(\pi_0, \pi_1, \theta)$  would vote for

$$\tau_{t+1}(\pi_1 - \pi_0, \theta) = \frac{\lambda(H_{t+1}) (\pi_1 - \pi_0)}{a (y_1 - y_0) \theta^2}$$

## Dynastic Learning

- Individuals have different beliefs about  $(\pi_0, \pi_1, \theta)$

- Agent  $i_t$  has beliefs  $\mu_t^i: \Pi_0 \times \Pi_1 \times \Theta \rightarrow [0, 1]$

- Assume that  $\Pi_0$ ,  $\Pi_1$  and  $\Theta$  are finite (not critical in any way, just helps exposition)

True parameters,  $(\pi_0^*, \pi_1^*, \theta^*)$ , are time-stationary

The state of the economy in period  $t$  can be summarized by  $(L_t, H_t, \tau_t, (\mu_t^i)_{i \in I})$

Learning technology is Bayesian, but there is no common knowledge of Bayesian rationality

- Assumption equivalent to the "strategic myopia" assumption in learning in games

By linearity, agent  $i_t$  will choose effort level  $e_t^i(\tau_t, \mu_t^i) = e(\tau_t, \theta_t^i)$  where

$$\theta_t^i = \mathbb{E}_{\mu_t^i}[\theta] = \sum_{(\pi_0, \pi_1, \theta) \in \text{supp}(\mu_t^i)} \theta \mu_t^i(\pi_0, \pi_1, \theta).$$

Assume agent  $i_t$  observes  $\hat{\theta}_t^{-i} = \int_{j \neq i} \theta_t^j dj$

$i_t$ 's most preferred tax rate is found by maximizing the following with respect to  $\tau$ :

$$\sum_{\substack{(\pi_0, \pi_1, \theta) \\ \in \text{supp}(\mu_t^i)}} \mu_t^i(\pi_0, \pi_1, \theta) \left[ \begin{array}{l} \left( \pi_0 + \theta e\left(\tau, \hat{\theta}_t^{-i}\right) \right) (1 - \tau) y_1 \\ + \tau y_0 - \frac{e\left(\tau, \hat{\theta}_t^{-i}\right)^2}{2a} \\ + \left( \begin{array}{l} 1 - \pi_0 \\ -\theta e\left(\tau, \hat{\theta}_t^{-i}\right) \end{array} \right) (1 - \tau) y_0 \\ + \tau \left( \begin{array}{l} \pi_0 \lambda (L_{t+1}) \\ + \pi_1 \lambda (H_{t+1}) \\ + \theta e\left(\tau, \hat{\theta}_t^{-i}\right) \end{array} \right) (y_1 - y_0) \end{array} \right]$$

The FOC for this problem implies:

$$\tau = \frac{(\pi_1^{i_t} - \pi_0^{i_t}) \lambda (H_{t+1})}{(y_1 - y_0) a (\hat{\theta}_t^{-i})^2} + \frac{\hat{\theta}_t^{-i} - \theta_t^i}{(\hat{\theta}_t^{-i})}$$

- Thus  $i_t$ 's preferred tax rate is

$$\tau_{t+1}^i(\mu_t^i, \hat{\theta}_t^{-i}) = \frac{\lambda (H_{t+1}) (\pi_1^{i_t} - \pi_0^{i_t})}{a (y_1 - y_2) (\hat{\theta}_t^{-i})^2} + 1 - \frac{\theta_t^i}{\hat{\theta}_t^{-i}}$$

Modelling of the political process is minimal

$$\tau_{t+1} = \text{med} \{ \tau_{t+1}^i(\mu_t^i, \theta_t^{-i}) : i \in [0, 1] \}$$

- This makes sense since preferences are single-peaked (we know that this will be the Condorcet winner)
- Follows because  $V_{t+1}$  is a quadratic in  $\tau$

Bayesian updating by an agent  $i_t$  in state  $(L_t, H_t, \tau_t, (\mu_t^i)_{i \in [0,1]})$  yields the following posterior beliefs:

$$\mu_{t+1}^i(\pi_0, \pi_1, \theta) = \frac{\Pr(y|\pi_0, \pi_1, \theta) \mu_t^i(\pi_0, \pi_1, \theta)}{\sum_{(\tilde{\pi}_0, \tilde{\pi}_1, \tilde{\theta}) \in \mathbf{S}(\mu_t^i)} \Pr(y|\tilde{\pi}_0, \tilde{\pi}_1, \tilde{\theta}) \mu_t^i(\tilde{\pi}_0, \tilde{\pi}_1, \tilde{\theta})},$$

where  $\mathbf{S}(\mu_t^i) = \text{supp}(\mu_t^i)$ , the support of  $\mu_t^i$

- For example, if  $y_{t-1}^i = y_x$  and  $y_t^i = y_1$ , then:

$$\mu_{t+1}^i(\pi_0, \pi_1, \theta) = \frac{\mu_t^i(\pi_0, \pi_1, \theta) [\pi_x + \theta e(\theta_t^i, \tau_t)]}{\sum_{(\tilde{\pi}_0, \tilde{\pi}_1, \tilde{\theta}) \in \mathbf{S}(\mu_t^i)} (\pi_x + \theta e(\theta_t^i, \tau_t)) \mu_t^i(\tilde{\pi}_0, \tilde{\pi}_1, \tilde{\theta})}$$

### Steady-State Political Attitudes

Note that beliefs are a Martingale since

$$\begin{aligned} \mathbb{E}_{\mu_t^i} [\mu_{t+1}^i(\pi_0, \pi_1, \theta)] &= \mathbb{E}_{\mu_t^i} \left[ \frac{\mu_t^i(\pi_0, \pi_1, \theta) \Pr(y|\pi_0, \pi_1, \theta)}{\mathbb{E}_{\mu_t^i} [\Pr(y|\tilde{\pi}_0, \tilde{\pi}_1, \tilde{\theta})]} \right] \\ &= \mu_t^i(\pi_0, \pi_1, \theta) \end{aligned}$$

Take  $(\pi_0, \pi_1, \theta) \neq (\pi'_0, \pi'_1, \theta')$  and consider  $l_t = \frac{\mu_t^i(\pi_0, \pi_1, \theta)}{\mu_t^i(\pi'_0, \pi'_1, \theta')}$ , i.e., the likelihood ratio

Note that  $l_t$  is a Martingale since

$$\begin{aligned} \mathbb{E}_{\mu_t^i} [l_{t+1}] &= \frac{\mathbb{E}_{\mu_t^i} \left[ \mu_t^i(\pi_0, \pi_1, \theta) \frac{\Pr(y|\pi_0, \pi_1, \theta)}{\mathbb{E}_{\mu_t^i} [\Pr(y|\tilde{\pi}_0, \tilde{\pi}_1, \tilde{\theta})]} \right]}{\mathbb{E}_{\mu_t^i} \left[ \mu_t^i(\pi'_0, \pi'_1, \theta') \frac{\Pr(y|\pi_0, \pi_1, \theta)}{\mathbb{E}_{\mu_t^i} [\Pr(y|\tilde{\pi}_0, \tilde{\pi}_1, \tilde{\theta})]} \right]} \\ &= \frac{\mu_t^i(\pi_0, \pi_1, \theta)}{\mu_t^i(\pi'_0, \pi'_1, \theta')} = l_t \end{aligned}$$

Recall Doob's Martingale Convergence Theorem

**Theorem** (Doob (1953)). *If  $\{X_t\}_{t=1}^\infty$  is a non-negative martingale (wrt itself) on probability space  $(\Omega, \mathcal{F}, P)$ , then  $\lim_{t \rightarrow \infty} X_t$  exists and is finite P-a.s.*

Note that  $\mu_{t+1}^i(\pi_0, \pi_1, \theta)$  is clearly non-negative and thus the theorem applies, **IF** we can construct the underlying probability space.

Not easy to define (see Easley and Kiefer (1989) for details)

Define  $P$  the probability measure essentially as follows:

- Fix a sequence of tax rates  $\{\tau_t\}_{t=0}^\infty$
- Dynasty  $i$  starts with prior  $\mu_0^i$  and calculates the probability of obtaining all possible beliefs given this prior, the sequence of tax rates and the optimal effort choices of its various generations (technically this would be defined by using finite sequences and then applying the Kolmogorov extension theorem)
- This gives us a probability measure,  $P_{\mu_0^i, \{\tau_t\}_{t=0}^\infty}$ , which is a function of the prior and the sequence of tax rates
- Technical issues if the set  $(\Pi_0, \Pi_1, \Theta)$  is not finite since the martingale of interest takes on values of probability measures on this space (need an integral generalized to infinite dimensional spaces, for example Bochner integral)

Proposition 1 in the paper states:

**Proposition.** *For any initial state  $(L_0, \tau_0, (\mu_0^i)_{i \in I})$  and every  $i \in I$ , the belief  $(\mu_0^i)$  converges w.p. 1 to some  $\mu_\infty^i(\cdot)$ . The tax rate also converges, i.e.,  $\tau_t \rightarrow \tau_\infty$ .*

*Proof.* For any sequence of tax rates  $\{\tilde{\tau}_t\}_{t=0}^\infty$  and each  $i$ , by Doob's Martingale convergence theorem, we have that  $\mu_0^i \rightarrow \mu_\infty^{i, \{\tilde{\tau}_t\}_{t=0}^\infty}(\cdot)$ ,  $P_{\mu_0^i}$ -a.e. The tax sequence that is relevant can be found sequentially, by setting  $\tilde{\tau}_0 = \tau_0$  to be the specified initial condition and then for  $t \geq 1$  defining:

$$\tau_{t+1} = \text{med} \left\{ \tau_{t+1}^i \left( \mu_t^i, \theta_t^{-i} \right) : i \in [0, 1] \right\}.$$

Then given this "realized" tax sequence  $\{\tau_t\}_{t=0}^\infty$ , we have that for each  $i$ ,  $\mu_0^i \rightarrow \mu_\infty^{i, \{\tau_t\}_{t=0}^\infty}(\cdot) \stackrel{\text{def}}{=} \mu_\infty^i(\cdot)$ . Since the tax rate is a continuous function of the beliefs, then  $\{\tau_t\}_{t=0}^\infty$  converges to some  $\tau_\infty$ .  $\square$

■ The above defines the steady state,  $(\tau_\infty, (\mu_\infty^i)_{i \in I})$ , which is a tax rate and belief profile pair

■ Note that  $L_0$  is not stable

■ For tax rate  $\tau \in [0, 1]$  let  $S(\tau)$  be the set of stable beliefs given  $\tau$ , i.e.  $\mu(\cdot) \in S(\tau)$  if

(a)  $(\pi_0^*, \pi_1^*, \theta^*) \in \text{supp}(\mu)$

(b)  $\pi_x + \theta e(\mu, \tau) = \pi_x^* + \theta^* e(\mu, \tau)$  for all  $x \in \{0, 1\}$  and  $(\pi_0, \pi_1, \theta) \in \text{supp}(\mu)$

■ This stable set is very much like self-confirming equilibrium in the learning in games literature

**Proposition.** For initial priors  $(\mu_0^i)_{i \in I}$  with  $\mu_0^i(\pi_0^*, \pi_1^*, \theta^*) > 0$  for all  $i$ , in the steady state we have  $\mu_\infty^i \in S(\tau_\infty)$  for all  $i$  and  $\tau_\infty = \text{med} \left\{ \tau_\infty^i \left( \mu_\infty^i, \widehat{\theta}_\infty^{-i} \right) : i \in I \right\}$ . Conversely, for any steady state  $(\tau_\infty, (\mu_\infty^i)_i)$  which is sensible, i.e.,  $\mu_\infty^i \in S(\tau_\infty)$  for all  $i$  and  $\tau_\infty = \text{med} \left\{ \tau_\infty^i \left( \mu_\infty^i, \widehat{\theta}_\infty^{-i} \right) : i \in I \right\}$ , there is a  $(\mu_0^i)_{i \in I}$  that converges to this steady state  $(\tau_\infty, (\mu_\infty^i)_i)$ .

*Proof.* ( $\Rightarrow$ ) We have to show that  $\mu_\infty^i \in S(\tau_\infty)$ . To see that property (b) holds if (a) holds, we note that by definition of stationarity  $i$  never changes the probability it assigns to anything in the support of  $\mu_\infty^i$  given any observation, so that for any  $(\pi_0, \pi_1, \theta), (\pi_0', \pi_1', \theta') \in \text{supp}(\mu_\infty^i)$ , we have that:

$$\pi_x + \theta e(\mu, \tau) = \pi_x' + \theta' e(\mu, \tau), \quad \forall x \in \{0, 1\}.$$

If (a) holds, then  $(\pi_0^*, \pi_1^*, \theta^*) \in \text{supp}(\mu_\infty^i)$  and (b) is satisfied.

Thus we are left to show that  $\mu_\infty^i(\pi_0^*, \pi_1^*, \theta^*) > 0$ . Assume by way of contradiction that  $\mu_\infty^i(\pi_0^*, \pi_1^*, \theta^*) = 0$ . Then there exists some  $(\pi_0, \pi_1, \theta)$ , such that  $\mu_\infty^i(\pi_0, \pi_1, \theta) > 0$ . Consider  $l_t = \frac{\mu_t^i(\pi_0, \pi_1, \theta)}{\mu_t^i(\pi_0^*, \pi_1^*, \theta^*)}$  and note that since it is a martingale,  $l_t \rightarrow l_\infty$  and  $l_\infty < \infty$  with probability 1 (probability  $P_{\mu_0^i, \{\tau_t\}_{t=0}^\infty}$ ). Thus,  $\mu_t^i(\pi_0^*, \pi_1^*, \theta^*) > 0$  a.s.. Finally, note that  $\tau_\infty$  is given by the specified formula since we are in a steady state.

( $\Leftarrow$ ) For any sensible steady state  $(\tau_\infty, (\mu_\infty^i)_i)$ , let the prior be the steady-state belief, i.e.,  $(\mu_0^i)_{i \in I} = (\mu_\infty^i)_{i \in I}$  and the initial tax rate be  $\tau_0 = \tau_\infty$ . Note that  $\mu_t^i = \mu_\infty^i$  for all  $i$  and all  $t$  and that  $\tau_t = \tau_\infty$  for all  $t$ .  $\square$

■ Bayesian learning does not converge to the truth (agents have no incentives for experimentation)

• In that sense, individual agents are infinitely impatient

■ However, predictions of the model are very consistent with the leading empirical observation

■ Note that agents who converge to higher  $\theta_\infty^i = \mathbb{E}_{\mu_\infty^i}[\theta]$  will put in more effort, as  $e(\tau_t, \theta_t^i)$  is increasing in  $\theta_t^i$ , and prefer lower taxes, as  $\tau_{t+1}^i$  is decreasing in  $\theta_t^i$

■ Let  $H_\infty(\theta) = \lambda(\{i : \theta_\infty^i = \theta \text{ and } y_\infty^i = y_1\})$  be the proportion of people who supply effort  $\theta = \theta_\infty^i$  and who have high income

■ If it is a steady state, we must have that this proportion is steady, so that

$$(\pi_0^* + \theta^* e(\tau_\infty, \theta))(1 - H_\infty(\theta)) = (1 - \pi_1^* - \theta^* e(\tau_\infty, \theta))H_\infty(\theta)$$

■ Solving the above gives us that in the steady state, we have

$$H_\infty(\theta) = \frac{\pi_0^* + \theta^* e(\tau_\infty, \theta)}{\pi_0^* - \pi_1^* + 1},$$

which is increasing in  $\theta$

■ We can then find steady state fractions of  $\theta$  dynasties in all the other partitions

$$\bullet UM_\infty(\theta) = (\pi_0^* + \theta^* e(\tau_\infty, \theta))(1 - H_\infty(\theta))$$

$$\bullet DM_\infty(\theta) = (1 - \pi_1^* - \theta^* e(\tau_\infty, \theta))$$

$$\bullet SH_\infty(\theta) = (\pi_1^* + \theta^* e(\tau_\infty, \theta))H_\infty(\theta)$$

$$\bullet SL_\infty(\theta) = (1 - \pi_0^* - \theta^* e(\tau_\infty, \theta))(1 - H_\infty(\theta))$$

■ For  $X \in \{H_t, L_t, SL_t, DM_t, UM_t, SH_t\}$  let  $X(\tau, \tau')$  be the proportion of agents in  $X$  who prefer  $\tau$  over  $\tau'$

**Proposition.** In the steady state, for  $\tau > \tau'$  we have (i)  $H_\infty(\tau, \tau') < L_\infty(\tau, \tau')$ , (ii)  $SH_\infty(\tau, \tau') < UM_\infty(\tau, \tau')$  and (iii)  $DM_\infty(\tau, \tau') < SL_\infty(\tau, \tau')$

*Proof.* Note that since preferences over taxes are single peaked, there exists some  $\tau'' \in (\tau', \tau)$  such that  $i \in I$  prefers  $\tau$  to  $\tau'$  if  $\tau_\infty^i(\cdot) \geq \tau''$ . Since  $\tau_\infty^i(\cdot)$  is decreasing in  $\theta_t^i = \mathbb{E}_{\mu_t^i}[\theta]$ , and everything is continuous then there exists some  $\theta''$  such that  $i \in I$  prefers  $\tau$  to  $\tau'$  if  $\theta_t^i < \theta''$ . Claim (i) follows from the fact that  $H_\infty(\theta)$  is increasing in  $\theta$ , which implies that the fraction of the high-income class with a  $\theta$  below some cutoff is smaller than the respective fraction of the low-income class. For claim (ii) note that the ratio:

$$\begin{aligned} \frac{SH_\infty(\theta)}{UM_\infty(\theta)} &= \frac{(\pi_1^* + \theta^* e(\tau_\infty, \theta))H_\infty(\theta)}{(\pi_0^* + \theta^* e(\tau_\infty, \theta))(1 - H_\infty(\theta))} \\ &= \frac{(\pi_1^* + \theta^* e(\tau_\infty, \theta))}{(1 - (\pi_1^* - \pi_0^*)) (1 - H_\infty(\theta))} \end{aligned}$$

is increasing with respect to  $\theta$  and thus the same argument follows. Finally, for claim (iii) we have that:

$$\begin{aligned} \frac{SL_\infty(\theta)}{DM_\infty(\theta)} &= \frac{(1 - \pi_0^* - \theta^* e(\tau_\infty, \theta))(1 - H_\infty(\theta))}{(1 - \pi_1^* - \theta^* e(\tau_\infty, \theta))H_\infty(\theta)} \\ &= \frac{1 - \pi_0^* - \theta^* e(\tau_\infty, \theta)}{\pi_0^* + \theta^* e(\tau_\infty, \theta)}, \end{aligned}$$

(after substituting for  $\lambda(H_\infty)$  as calculated below), which is decreasing with respect to  $\theta$ .  $\square$

**Remark.** Furthermore, we can note that:

$$\frac{SH_\infty(\theta)}{DM_\infty(\theta)} = \frac{(\pi_1^* + \theta^* e(\tau_\infty, \theta))}{(1 - \pi_1^* - \theta^* e(\tau_\infty, \theta))},$$

is increasing in  $\theta$  and that

$$\frac{SL_\infty(\theta)}{UM_\infty(\theta)} = \frac{(1 - \pi_0^* - \theta^* e(\tau_\infty, \theta))}{(\pi_0^* + \theta^* e(\tau_\infty, \theta))},$$

is decreasing in  $\theta$ . This then implies that  $SH_\infty(\tau, \tau') < DM_\infty(\tau, \tau')$  and that  $UM_\infty(\tau, \tau') < SL_\infty(\tau, \tau')$  for  $\tau > \tau'$ .

## Concluding Remarks

- Piketty (1995) provides a model to understand an important stylized fact about redistribution, i.e. voters with identical incomes but different social origins vote differently
- Furthermore, Piketty (1995) shows that even when voters have the same objectives they can prefer different levels of redistribution if they have different beliefs about the level of ex-ante income persistence and how redistribution impacts incentives to work

## Comments on Typos

- Beware of abuse of notation,  $L_t = m(\{i : y_{it-1} = y_0\})$ , i.e., the measure, and  $L_t = \{i : y_{it-1} = y_0\}$ , i.e., the set
- Equation on top of page 564 should read:

$$\tau_{it}(\mu_{it}) = \frac{H_t(\pi_1(\mu_{it}) - \pi_0(\mu_{it}))}{a(y_1 - y_0)\theta_t^2} + 1 - \frac{\theta(\mu_{it})}{\theta_t}$$

- The equations on the bottom of page 569 should read:

$$\begin{aligned} (\pi_0^* + \theta^* e(\tau_\infty, \theta)) L_\infty(\theta) &= (1 - \pi_1^* - \theta^* e(\tau_\infty, \theta)) H_\infty(\theta), \\ H_\infty(\theta) &= \frac{\pi_0^* + \theta^* e(\tau_\infty, \theta)}{1 - (\pi_1^* - \pi_0^*)} \end{aligned}$$